1. *First subroutine UEXPAN is called*
   a. *Density and temperature dependent elastic properties are read from the .inp file, for VM9 and Zirconia, which are defined by the user. Specific heat for zirconia are also user defined.*
   b. *If the material is "VM9" then:*
      i. *Using Narayanswami form, ratio between relaxation time and reference (volume) relaxation time is determined as:*

$$\frac{\tau_i}{\tau_{is}} = \exp\left[-\frac{\Delta H}{R}\left(\frac{1}{T_{ref}} - \frac{x}{T} - \frac{1-x}{T_f}\right)\right]$$

$where, \tau_i = structural\ relaxation\ time$

$\tau_{is} = volume\ relaxation\ time,$ *which is taken as 1/10th of shear relaxation times*

$H = activation\ energy$

$x = experimentally\ defined\ constant$

$T_{ref} = reference\ temperature$

$T = current\ temperature\ at\ end\ of\ increment$

$T_f = fictive\ temperature$

      ii. *The partial fictive temperatures* $T_{fi}(t)$, *the fictive temperature* $T_f(t)$ *and the difference between fictive temperatures* ($dT_f$) *of two sequential steps are stored as state variables and calculated as:*

$$T_{fi}(t) = \frac{T_{fi}(t) + T * (\frac{\Delta t}{\tau_i})}{1 + (\frac{\Delta t}{\tau_i})}$$

$where, \Delta t = time\ increment$

$T_f(t) = \sum c_i T_{fi}(t)$

$where, c_i\ are\ the\ constants\ defined\ at\ the\ reference\ temp$

$dT_f = T_f - T_f(previous\ increment)$

      iii. *The thermal contraction coefficients for glass and liquid states respectively, are obtained as:*

$$a_g = a_{g1} + a_{g2} * T + a_{g3} * T^2$$
$$a_{gf} = a_{g1} + a_{g2} * T_f + a_{g3} * T_f^2$$
$$a_{lf} = a_{l1} + a_{l2} * T_f + a_{l3} * T_f^2$$

*Where,* $a_{g1}, a_{g2}, a_{g3}, a_{l1}, a_{l2}$ *are the constants for thermal contraction coefficients in which subscript g is for glass and l for liquid state*

      iv. *If the current temperature is greater than 600 then,*
      $a_{lf}$ *=2.534e-05*
      *else if it is less than 500 then,*
      $a_{lf}$ *= 1.082e-05.*
      v. *Finally coefficient of expansion for VM9 is calculated as:*

$$expansion = a_g * temp.increment + (a_{lf} - a_{gf}) * fictive\ temp.increment$$

   c. *Else if the material is "Zirconia" then:*
      i. *If the temperature is less than 500 then,*

$$a_g = 1.025e-05$$

      *else if temperature is greater than 150 then,*
      *The thermal contraction coefficient for glass is obtained as:*

$$a_g = a_{g1} + a_{g2} * T + a_{g3} * T^2$$

$where, a_{g1}, a_{g2}, a_{g3}$ $are\ constants\ for\ thermal\ contraction\ coefficients\ for\ glass$

ii. $expansion = a_g * temp.increment$

2. *After ending UEXPAN subroutine, UTRS is called to account for the temperature-time shift for a time domain viscoelastic analysis*
   a. *As VM9 is the only viscoelastic material, Prony series for VM9 is defined by the user in the .inp file and read when required.*
   b. *x, an experimentally defined constant, is taken as 0.27 and activation energy* $\Delta H$, *reference temperature* $T_{ref}$ *along with ideal gas constant* $R$ *are defined*
   c. *Fictive temperature* ($T_f$) *and the difference between current and previous increment fictive temperatures* ($dT_f$) *are called from UEXPAN subroutine as statev.*
   d. *The shift along the horizontal axis, shift factor, are determined using Tool-Narayanswamy form. The shift factor is determined at the beginning and end of the increment using:*

$$shift = \exp\left[-\frac{\Delta H}{R}\left(\frac{1}{T_{ref}} - \frac{x}{T(\acute{t})} - \frac{1-x}{T_f(\acute{t})}\right)\right]$$

$where, T(\acute{t})\ is\ the\ temperature\ at\ any\ time\ \acute{t}$

$T_f(\acute{t}) is\ the\ fictive\ temperature\ at\ \acute{t}$